

Statistics

Spring 2023

Lecture 30



Feb 19-8:47 AM

Suppose You are taking an exam with 150 questions.
Each question has 6 choices, and only one correct choice per question.

Assume You are making random guesses and Success is to guess a correct ans.

$$1) n = 150 \quad 2) P = \frac{1}{6} \quad 3) q = 1 - P = \frac{5}{6}$$

$$4) \mu = np = 150 \left(\frac{1}{6}\right) = 25 \quad 5) \sigma^2 = npq = 150 \left(\frac{1}{6}\right) \left(\frac{5}{6}\right) = 20.83 = \frac{125}{6}$$

$$6) \sigma = \sqrt{\sigma^2} = \sqrt{\frac{125}{6}} = 4.564$$

Round μ & σ to a whole #, then find

$$\begin{array}{c} \mu = 25 \\ \sigma = 5 \end{array}$$

7) 68% Range

$$\mu \pm \sigma = 25 \pm 5 \Rightarrow 20 \text{ to } 30$$

8) Usual Range

95% Range

$$\mu \pm 2\sigma = 25 \pm 2(5)$$

$$= 25 \pm 10$$

$$\Rightarrow 15 \text{ to } 35$$

Apr 10-7:16 AM

Find the prob. that we guess

9) exactly 30 correct answers.

$$P(X=30) = \text{binompdf}(150, 1/6, 30) = .046$$

10) fewer than 30 correct answers.

$$P(X < 30) = P(X \leq 29) = \text{binomcdf}(150, 1/6, 29) = .838$$

11) more than 30 correct answers.

$$P(X > 30) = P(X \geq 31) = 1 - P(X \leq 30) = 1 - \text{binomcdf}(150, 1/6, 30) = .116$$

12) between 15 and 35 correct answers, inclusive.

$$P(15 \leq X \leq 35) = \text{binomcdf}(150, 1/6, 35) - \text{binomcdf}(150, 1/6, 14) = .979$$

Reduce by 1

Apr 10-7:24 AM

Geometric Prob. dist. SG 17

It is similar to binomial prob. dist. but

1) there is no n .

2) x is the number where first success happens.

3) $p \rightarrow$ Prob. of Success, $q \rightarrow$ Prob. of failure

$$p + q = 1$$

$$q = 1 - p$$

$$P(X) = p \cdot q^{x-1} \quad x = 1, 2, 3, 4, \dots$$

Suppose we have a geometric Prob. dist

with $p = .4$

$$q = 1 - p = .6$$

$P(\text{Success happens on 3rd trial})$

$$= P(X=3) = (.4)(.6)^{3-1} = .4(.6)^2 = .144$$

using TI

2nd | VARS

geometpdf(4,3)

$P=.4$

$X=3$

Paste

Apr 10-7:36 AM

Suppose you have .3 prob. to make a basket per shot in a basketball game.

$$P = .3 \quad q = 1 - P = .7$$

$P(\text{make first basket on the 4th shot})$


$$= P(X=4) = \text{geomet pdf}(.3, 4) = \boxed{.103}$$

$P(\text{first made basket happens by the 4th shot})$

$$= P(X \leq 4) = \text{geomet cdf}(.3, 4) = \boxed{.760}$$

$P(\text{first made basket happens after the 2nd shot})$

$$= P(X > 2) = P(X \geq 3) = 1 - P(X \leq 2) = 1 - \text{geomet cdf}(.3, 2)$$



$$= \boxed{.49}$$

Apr 10-7:45 AM

Mean $\mu = \frac{1}{p}$

Variance $\sigma^2 = \frac{q}{p^2}$

Standard deviation $\sigma = \sqrt{\sigma^2}$

Geometric
Prob.
Dist.

Consider a geometric Prob. dist. with $p = .2$

$$\mu = \frac{1}{p} = \frac{1}{.2} = \boxed{5} \quad q = 1 - p = \boxed{.8}$$

$$\sigma^2 = \frac{q}{p^2} = \frac{.8}{.2^2} = \boxed{20} \quad \sigma = \sqrt{\sigma^2} = \sqrt{20} \approx 4.5$$

$$P(X=2) = \text{geomet pdf}(.2, 2) = \boxed{.16}$$

$$P(X=3 \text{ or } X=5) = \text{geomet pdf}(.2, 3) + \text{geomet pdf}(.2, 5) = \boxed{.210}$$

$$P(X \leq 3) = \text{geomet cdf}(.2, 3) = \boxed{.488}$$

$$P(X \geq 3) = 1 - \text{geomet cdf}(.2, 2) = \boxed{.64}$$

Apr 10-7:54 AM

Poisson Prob. Dist.

- 1) x is # of Successes in a fixed interval
- 2) $x = 0, 1, 2, 3, \dots$
- 3) μ is the average # of Successes in that fixed interval.

$$P(x) = \frac{\mu^x}{x!} e^{-\mu}, \quad e \approx 2.718$$

Consider a poisson prob. dist. with $\mu=4$ in a fixed interval.

$$P(x=3) = \frac{4^3}{3!} \cdot e^{-4} = \frac{64}{6} \cdot (2.718)^{-4} \approx \boxed{.195}$$

Now using TI

2nd VARS Poisson Pdf(4,3) = .195

Lambda $\rightarrow \lambda = \mu = 4$

$x = 3$

Paste

Apr 10-8:04 AM

Suppose you work at a fast food and you get 25 orders in average per shift for pickup.

Fixed interval \rightarrow Per shift

Average in the fixed interval $\rightarrow \mu = 25$

$$P(\text{You get 30 orders}) = P(x=30)$$

$$= \text{Poisson.pdf}(25, 30)$$

$$= \boxed{.045}$$

$$P(\text{You get at most 30 orders})$$

$$P(x \leq 30) = \text{Poisson.cdf}(25, 30) = \boxed{.863}$$

$$P(\text{You get at least 30 orders})$$

~~29 30~~

$$= P(x \geq 30) = 1 - P(x \leq 29) = 1 - \text{Poisson.cdf}(25, 29) = \boxed{.182}$$

Apr 10-8:14 AM

Mean μ is given

Variance $\sigma^2 = \mu$

Standard deviation $\sigma = \sqrt{\sigma^2}$

} Poisson
} Prob.
} dist.

Suppose in average, I have 16 students absent in all my classes per week.

Per week \rightarrow Fixed interval SG 17

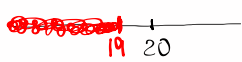
$\mu = 16$

$\sigma^2 = \mu = 16$ $\sigma = \sqrt{\sigma^2} = \sqrt{16} = 4$

Usual Range $\Rightarrow \mu \pm 2\sigma = 16 \pm 2(4) \Rightarrow 8 \text{ to } 24$

$P(\text{exactly } 20 \text{ absences}) = P(X=20)$
 $= \text{Poisson PDF}(16, 20) = 0.056$

$P(\text{fewer than } 20 \text{ absences}) = P(X < 20) = P(X \leq 19)$
 $= \text{Poisson CDF}(16, 19) = 0.812$



Apr 10-8:21 AM